A Simple n-Dimensional Intrinsically Universal Quantum Cellular Automata

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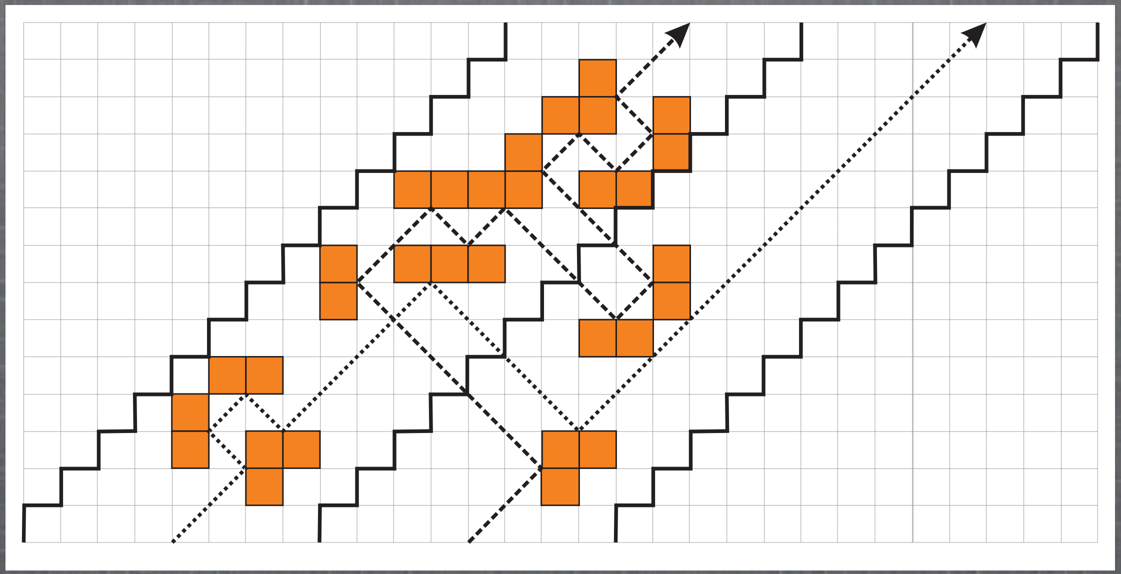
Pablo Arrighi



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Cellular Automata

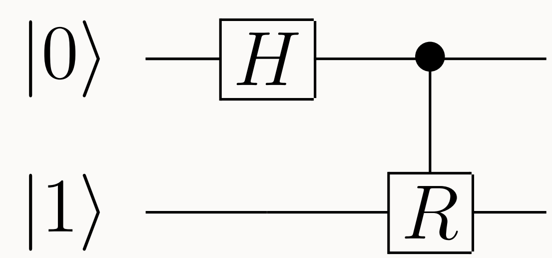
- A cellular automata is an array of identical cells
- Each takes one of a finite set of states
- Evolves by iteration of a global evolution function
- Defined over a local neighbourhood of cells, shift invariant
- Applied synchronously across grid

Quantum Computing

- Quantum bits:

$$|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle, \qquad |\alpha|^2 + |\beta|^2 = 1, \qquad \alpha, \beta \in \mathbb{C}$$

- Universal quantum circuits:



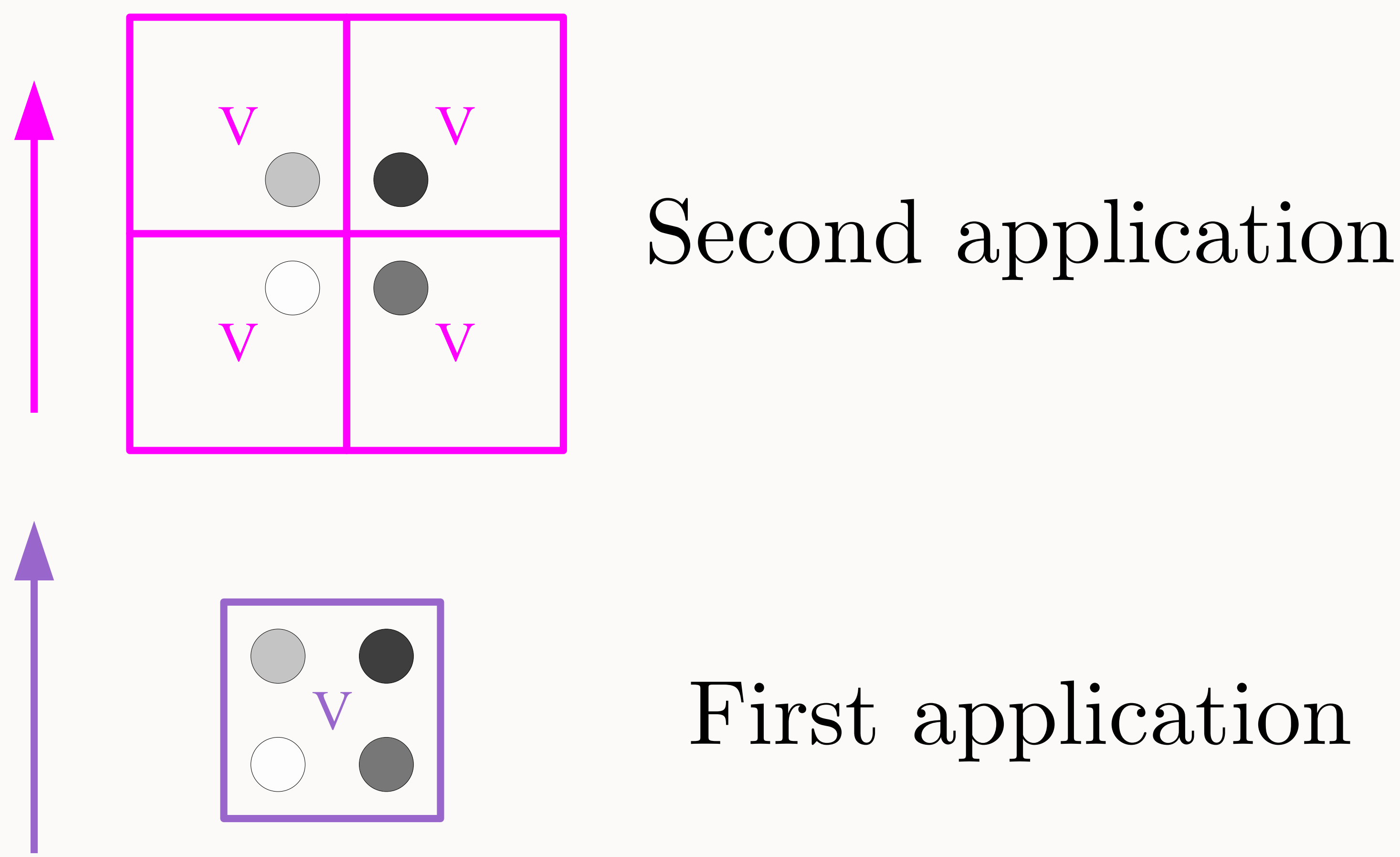
- ***H****adamard,* ***R****otation,* Controlled-Rotation

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad H^2 = \left(\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\right)^2 = \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

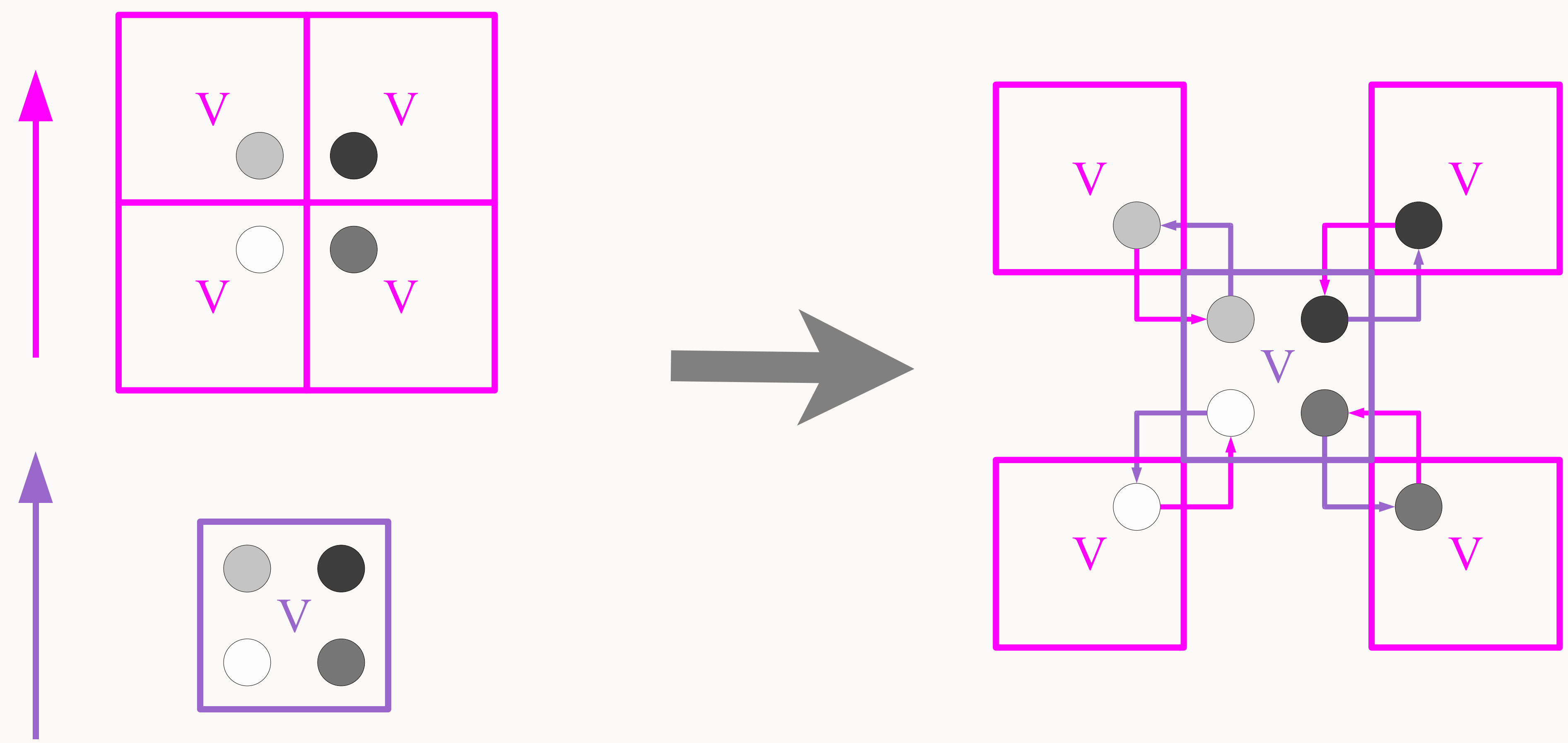
Introduction

- A simple Partitioned Quantum Cellular Automata
- Simulates any other *PQCA* in a topology preserving manner
- Able to simulate multiple iterations.
- Intrinsically universal
- Construction presented in 2D

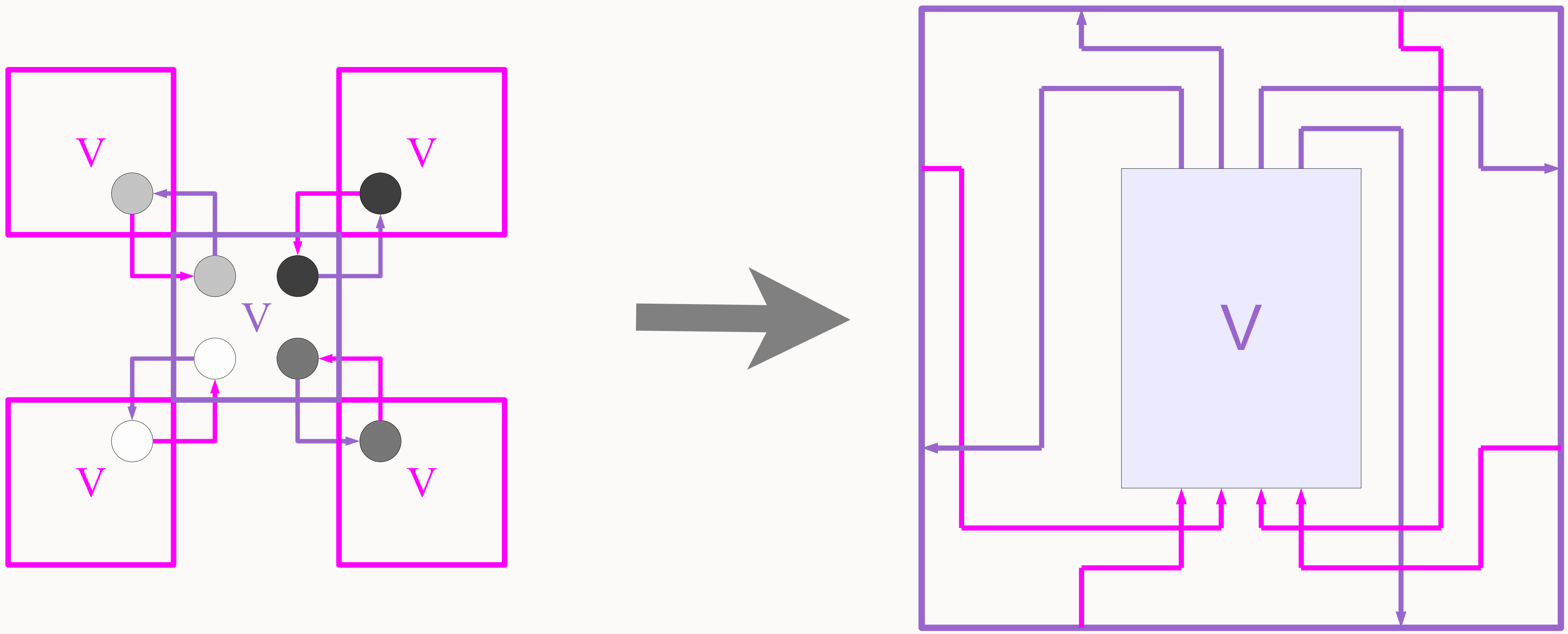
Partitioned QCA



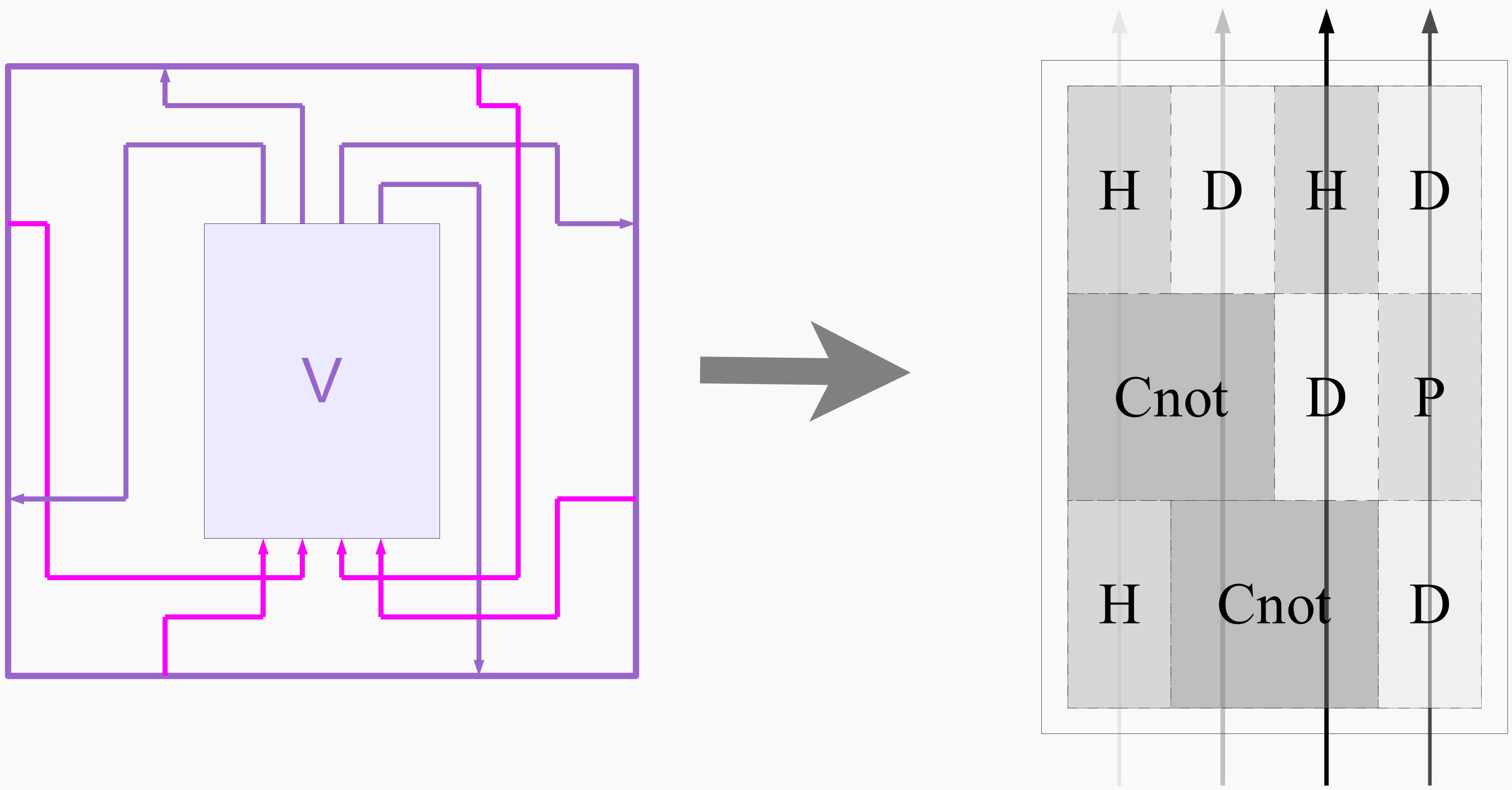
Flattening a PQCA



Flattening a PQCA

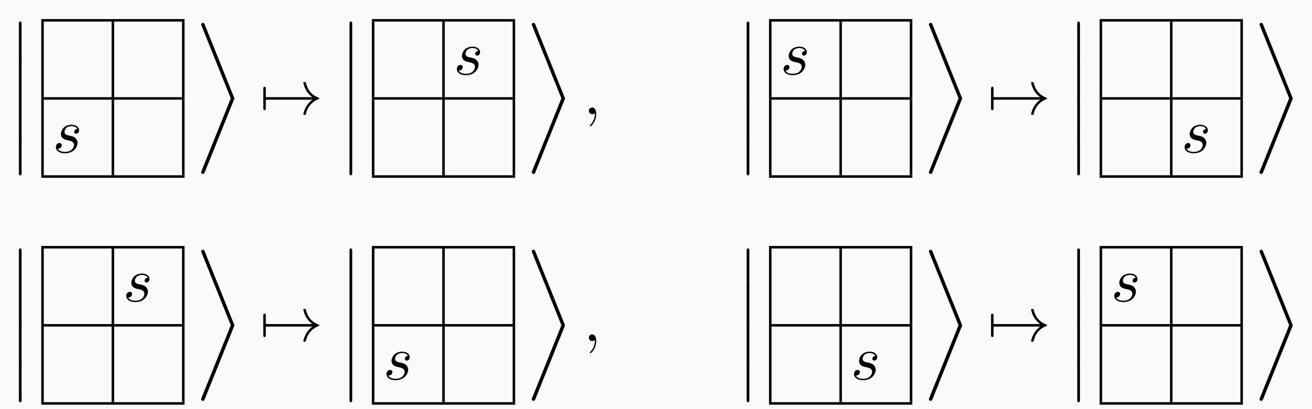


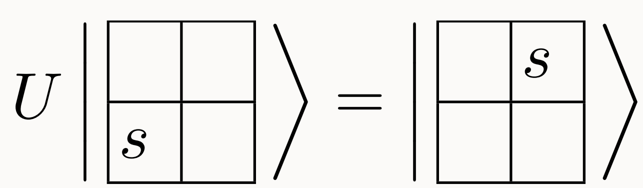
Flattening a PQCA



Signals

- Cells: *Blank, Qubit* (0 or 1) or a *Barrier*
- Signals carry qubits
- Diagonal propagation

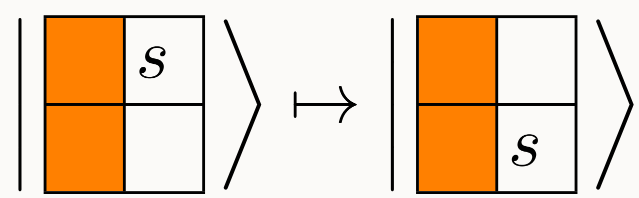




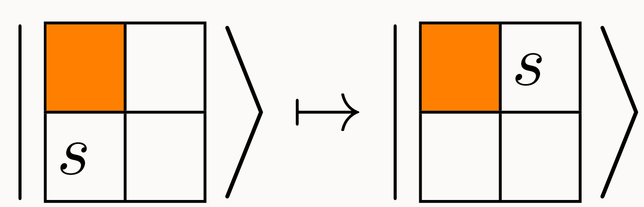
$s \in \{0, 1\}$

Barriers

- Two barriers: Signal redirection



- One barrier: No effect



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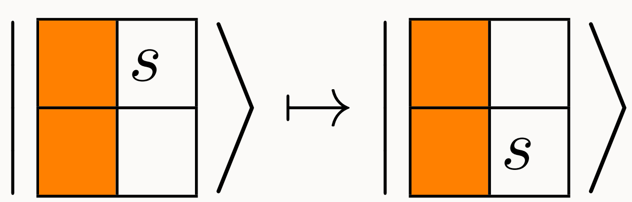

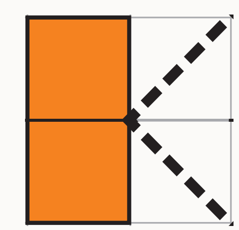

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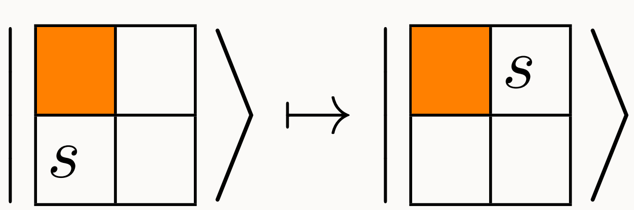
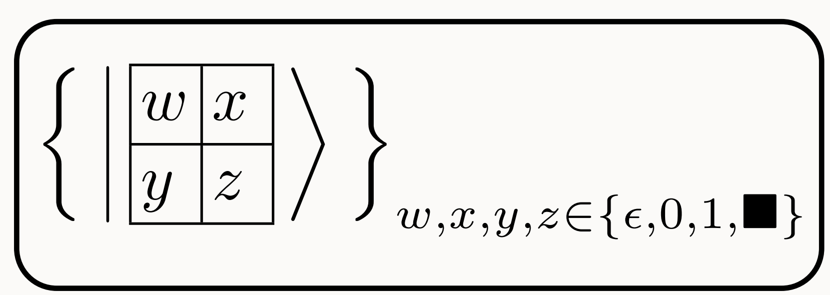
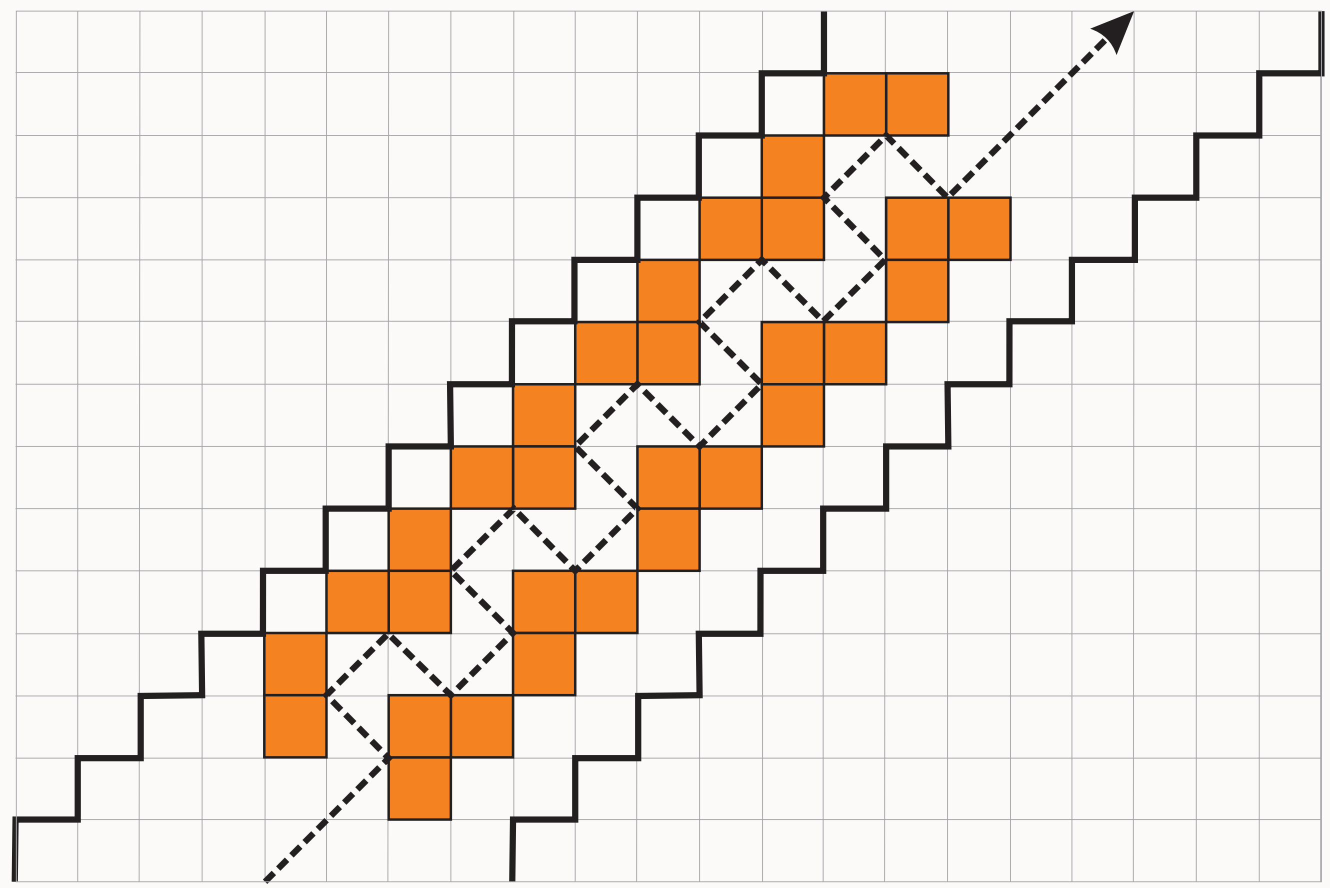




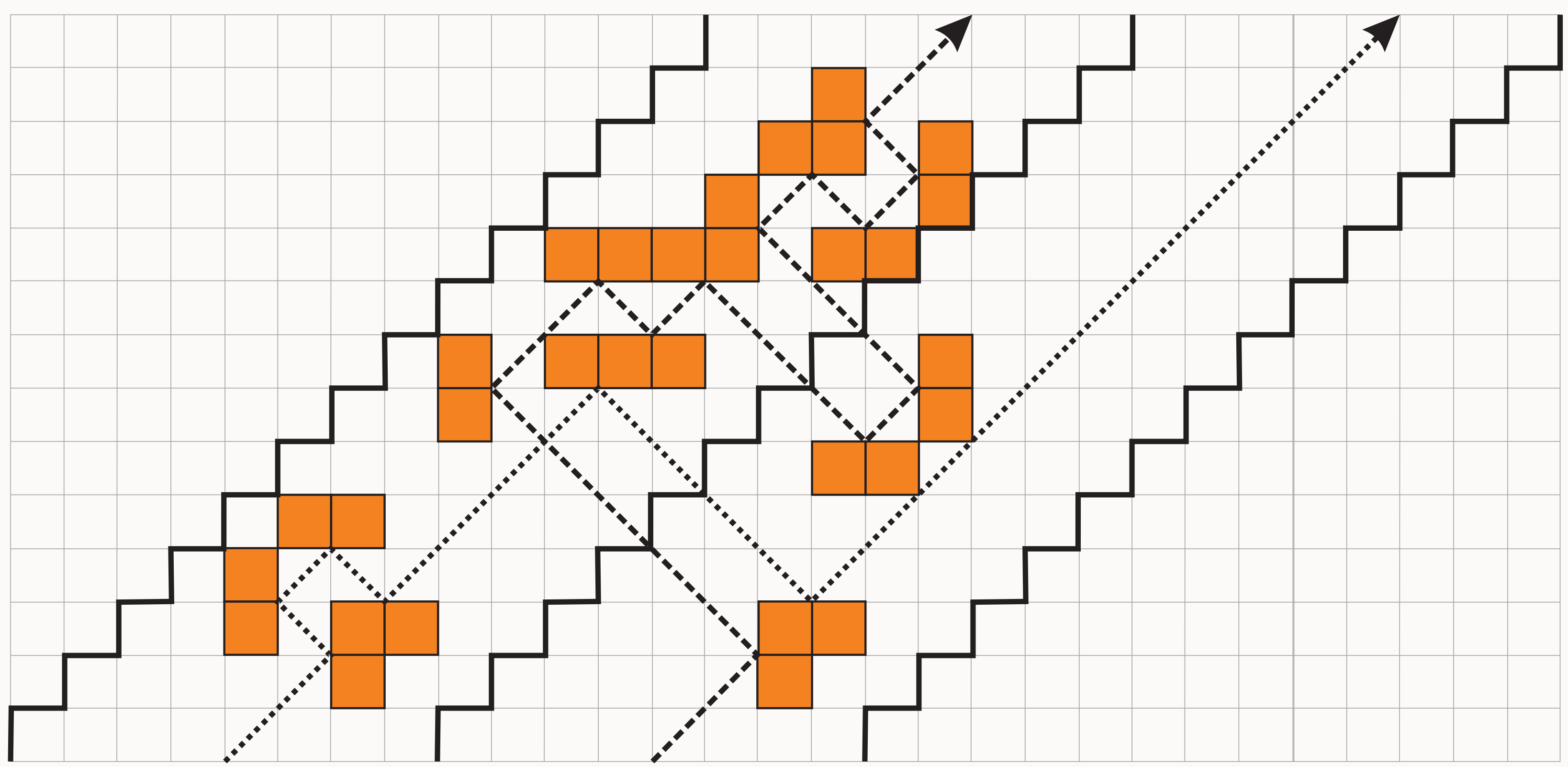
Identity (Delay) Circuit

- 8 x 14 tile
- 24 time steps



Swap Circuit

- 16 x 14 tile
- 24 time steps



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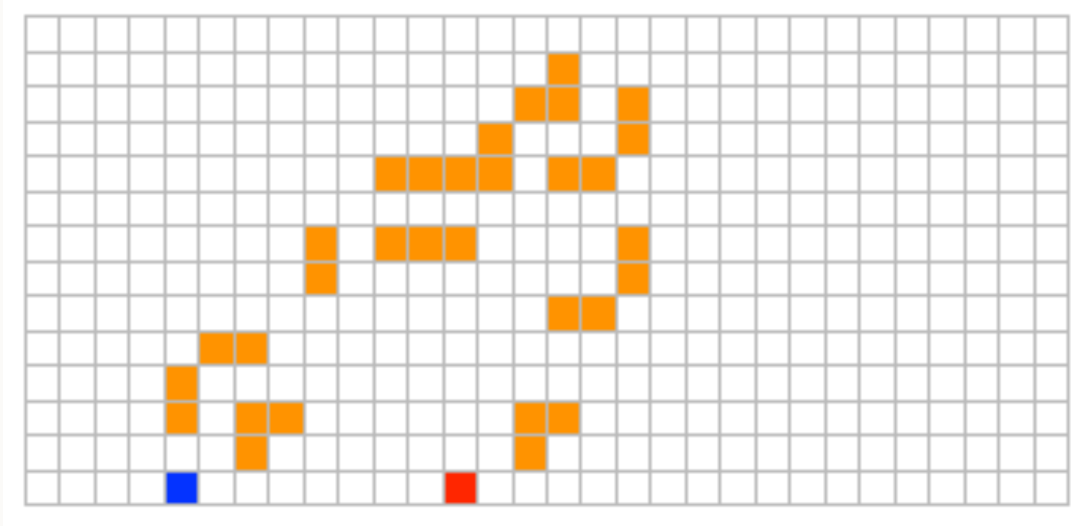
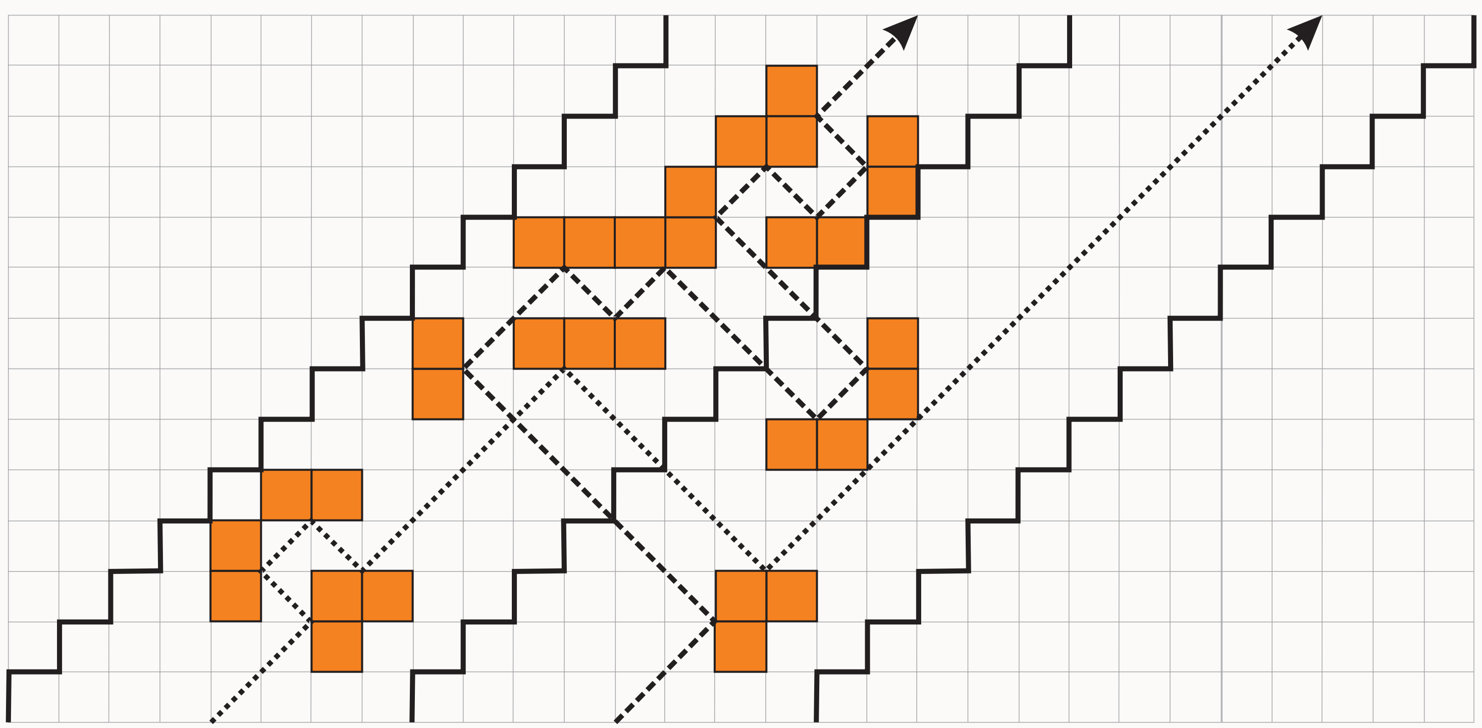
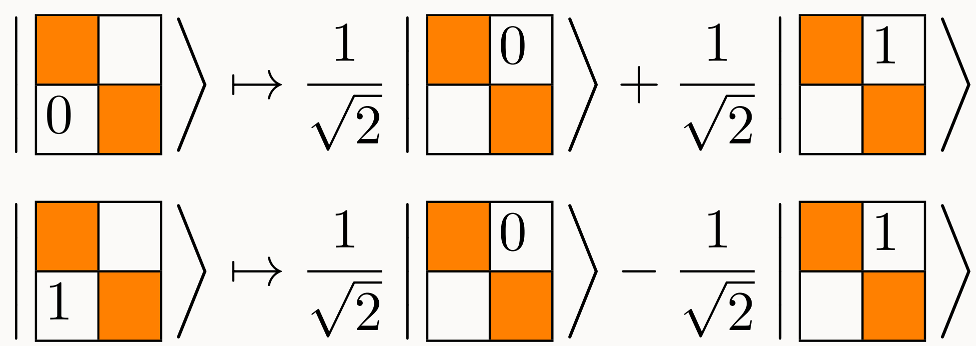




Quantum Gates

- Hadamard operation



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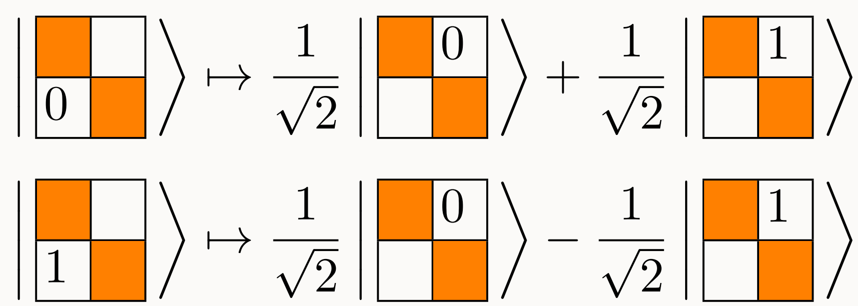

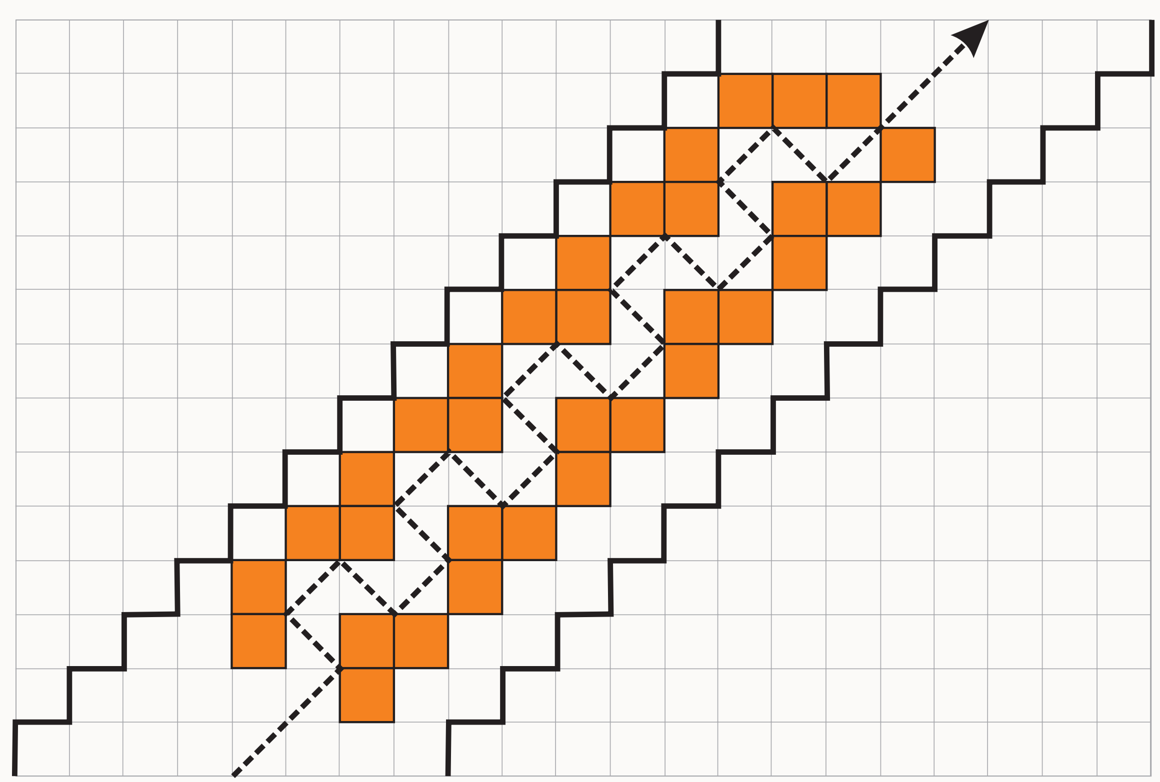

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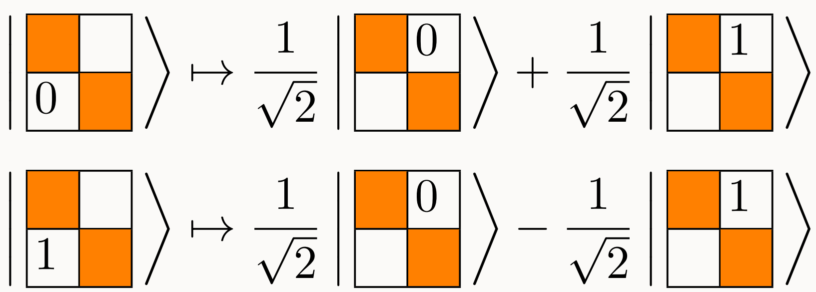
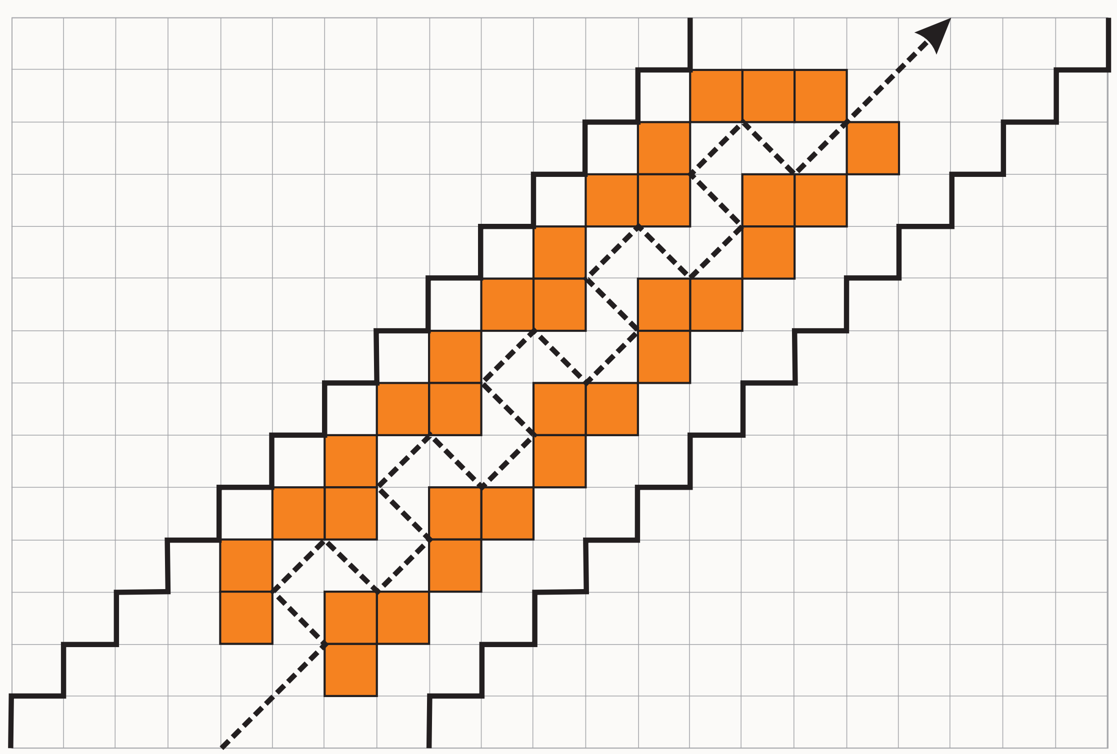
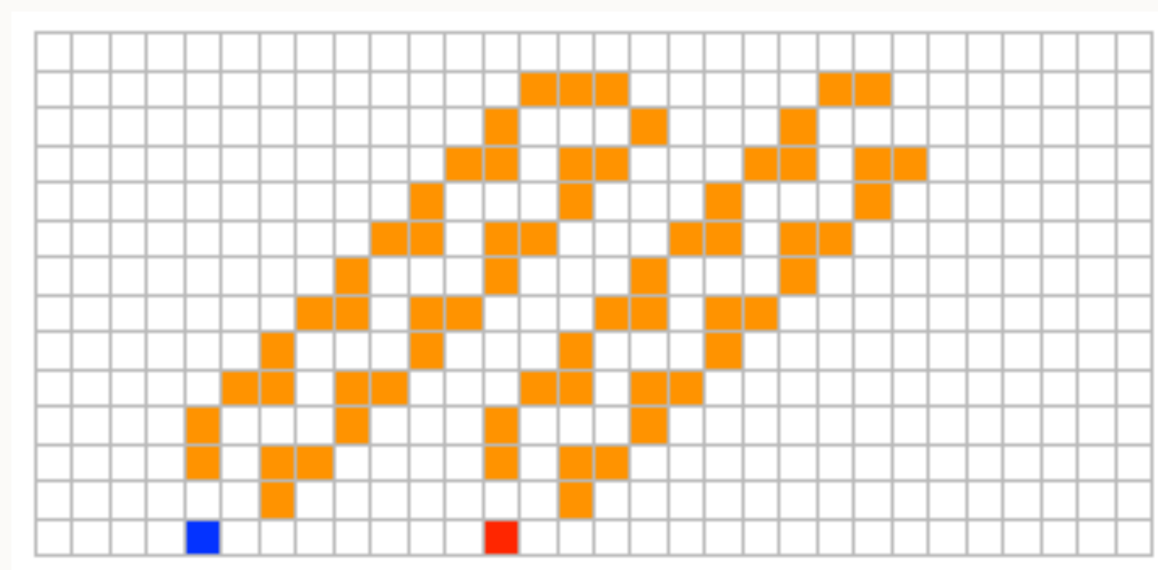






Quantum Gates II

- Signal collisions: Controlled-R(π/4)

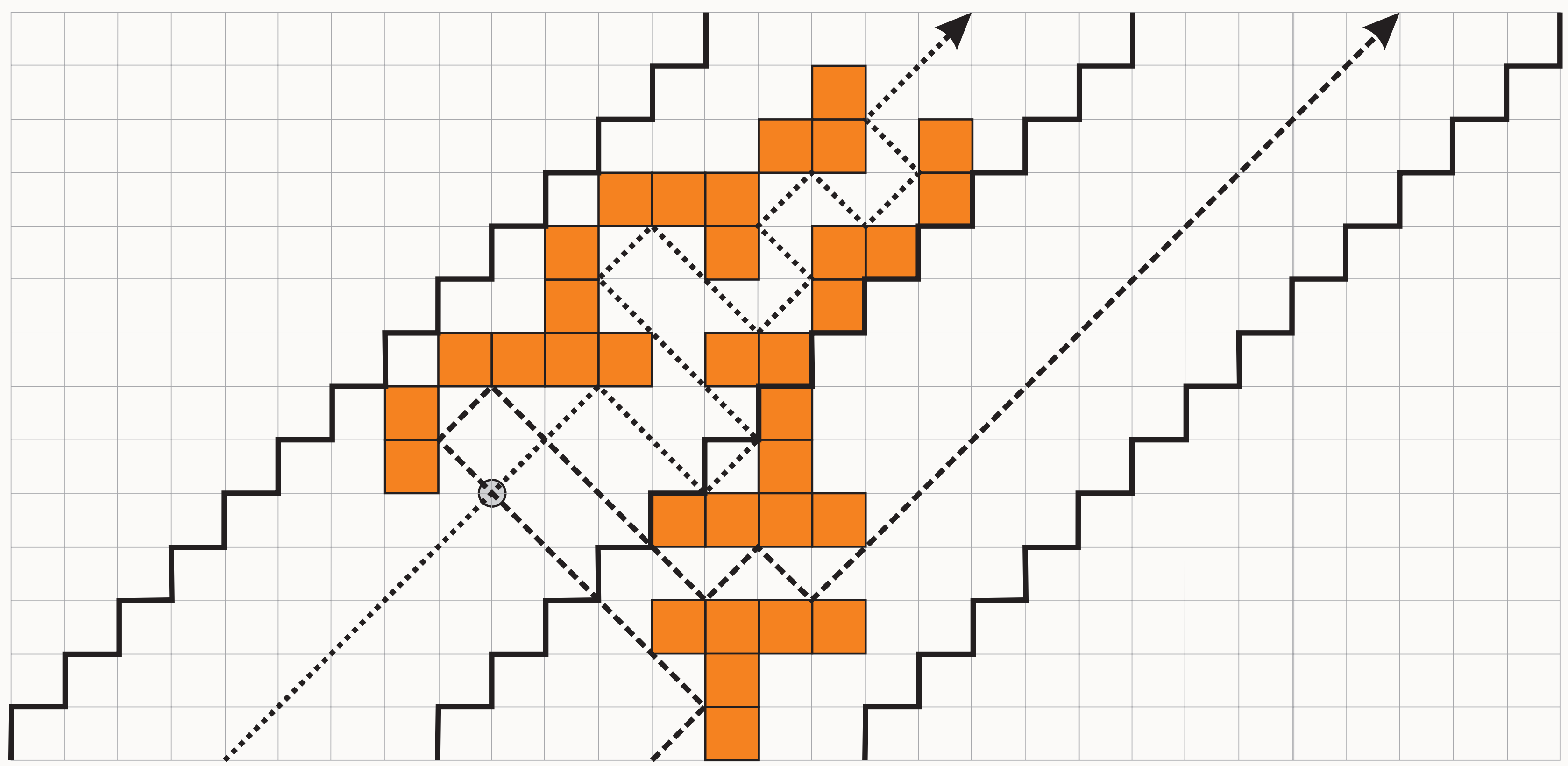
$$\left|\begin{array}{|c|c|}\hline 1 & \\ \hline 1 & \\ \hline\end{array}\right\rangle \mapsto e^{\frac{i\pi}{4}} \left|\begin{array}{|c|c|}\hline & 1 \\ \hline & 1 \\ \hline\end{array}\right\rangle, \qquad \left|\begin{array}{|c|c|}\hline x & \\ \hline y & \\ \hline\end{array}\right\rangle \mapsto \left|\begin{array}{|c|c|}\hline & y \\ \hline & x \\ \hline\end{array}\right\rangle otherwise$$

- Used to create two circuits:
  - Controlled-R(π/4)
  - R(π/4)

$$\left\{ \left|\begin{array}{|c|c|}\hline x & \\ \hline y & \\ \hline\end{array}\right\rangle \right\}_{x,y\in\{0,1\}}$$

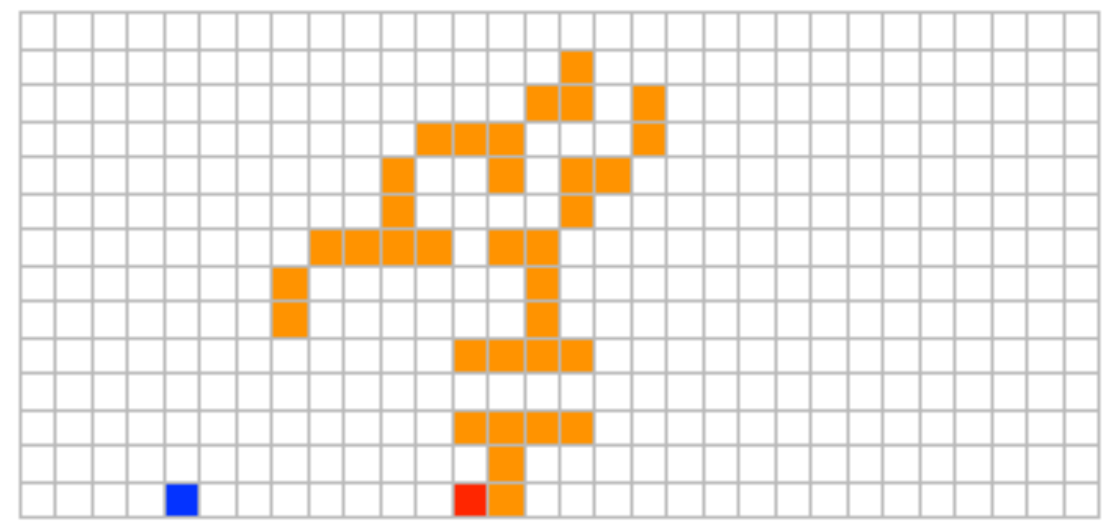
Controlled-R(π/4)

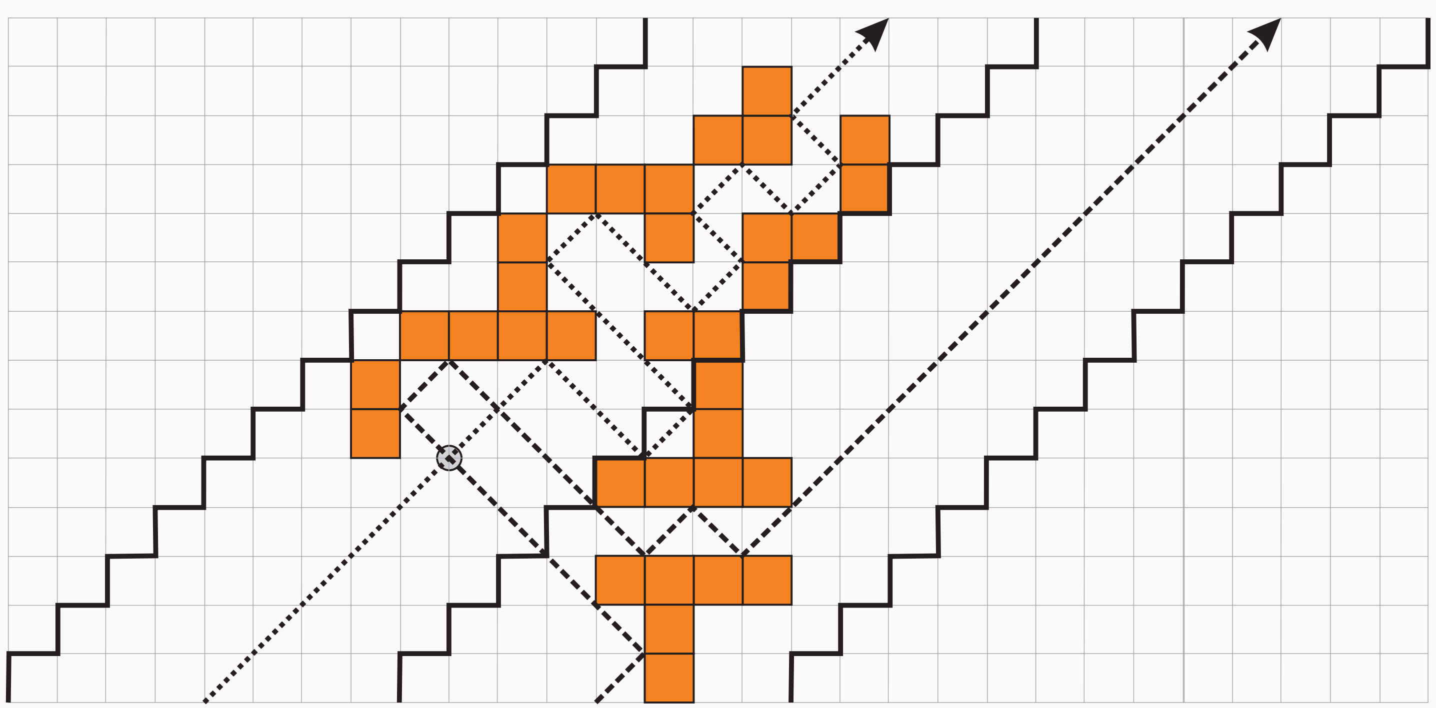
$$\left|\begin{array}{|c|c|}\hline 1 & \\ \hline 1 & \\ \hline\end{array}\right\rangle \mapsto e^{\frac{i\pi}{4}}\left|\begin{array}{|c|c|}\hline & 1 \\ \hline & 1 \\ \hline\end{array}\right\rangle, \qquad \left|\begin{array}{|c|c|}\hline x & \\ \hline y & \\ \hline\end{array}\right\rangle \mapsto \left|\begin{array}{|c|c|}\hline & y \\ \hline & x \\ \hline\end{array}\right\rangle \; otherwise$$



Controlled-R(π/4)

$$\left|\begin{array}{|c|c|}\hline 1 & \\ \hline 1 & \\ \hline\end{array}\right\rangle \mapsto e^{\frac{i\pi}{4}} \left|\begin{array}{|c|c|}\hline & 1 \\ \hline & 1 \\ \hline\end{array}\right\rangle, \qquad \left|\begin{array}{|c|c|}\hline x & \\ \hline y & \\ \hline\end{array}\right\rangle \mapsto \left|\begin{array}{|c|c|}\hline & y \\ \hline & x \\ \hline\end{array}\right\rangle \textit{otherwise}$$

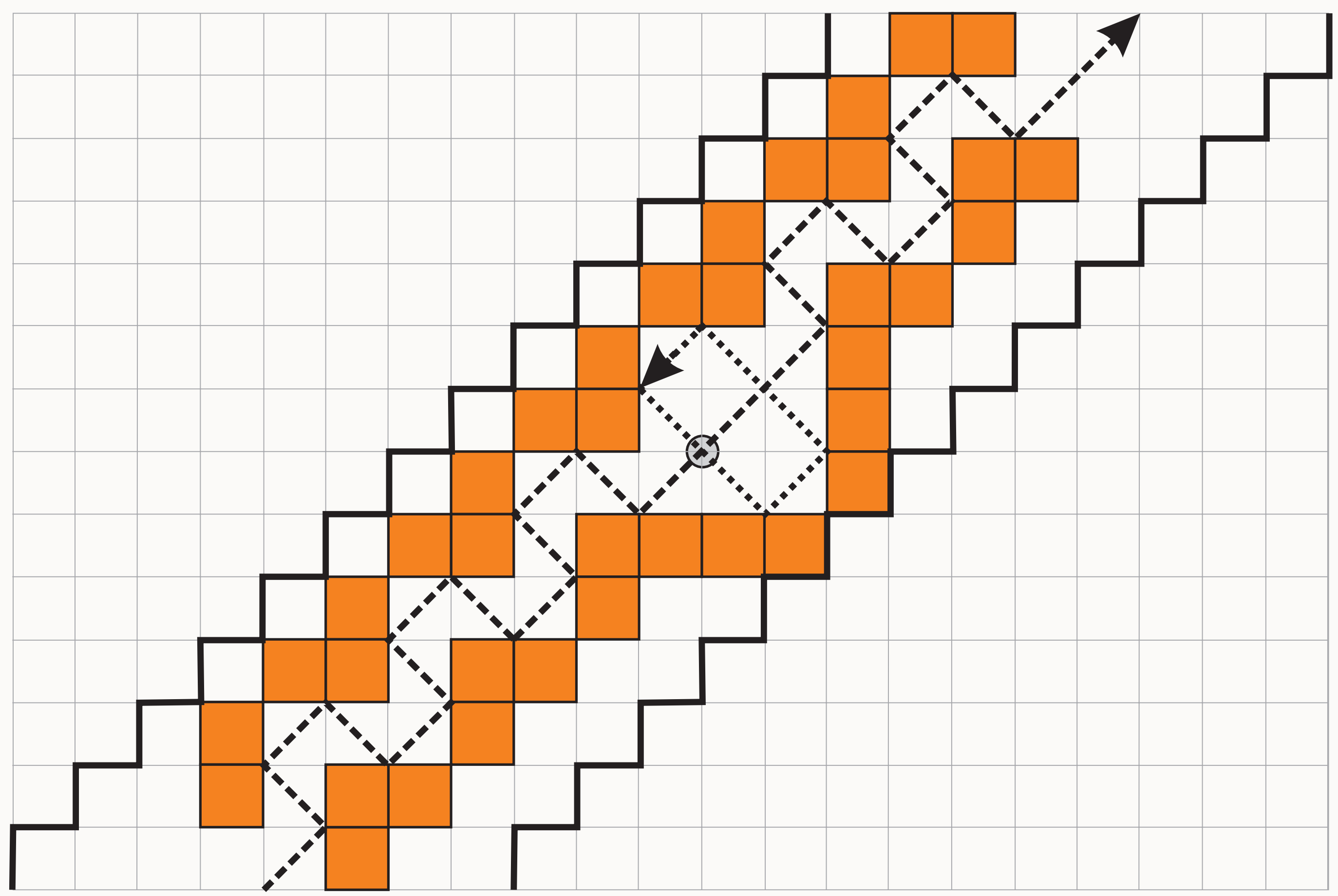




R(π/4) gate

- Auxiliary signal, set to 1
- Loops every 6 time-steps

$$|0\rangle \rightarrow |0\rangle$$
$$|1\rangle \rightarrow e^{\frac{i\pi}{4}}|1\rangle$$



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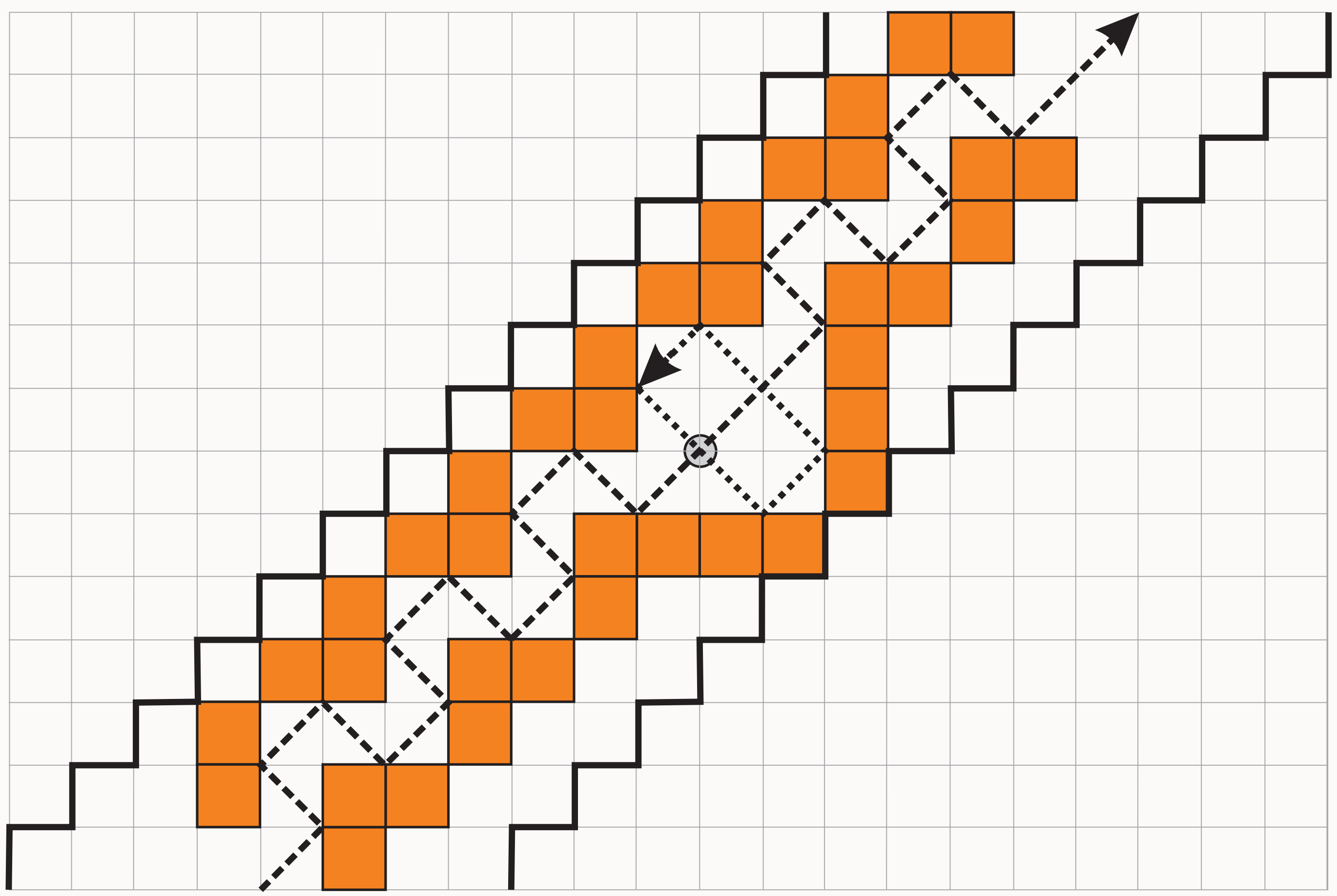
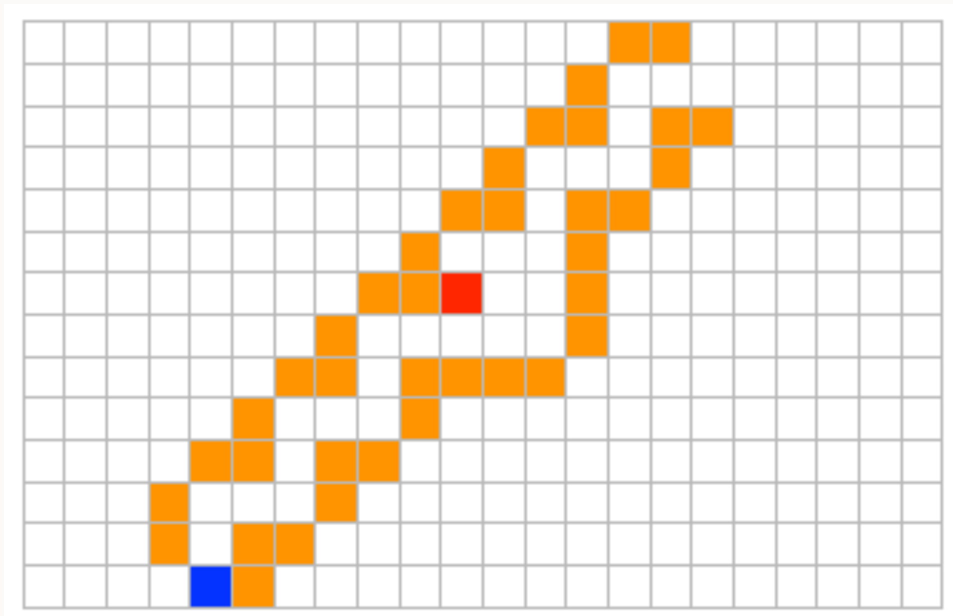
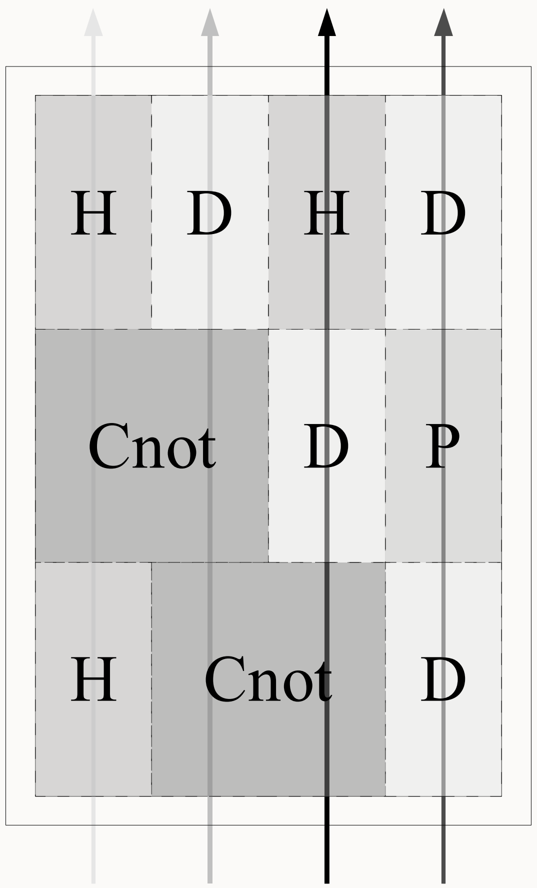




$$|0\rangle \rightarrow |0\rangle$$
$$|1\rangle \rightarrow e^{\frac{i\pi}{4}}|1\rangle$$

Circuits and wires

- Gates are modular, so can be combined.
- Each *qubit* occupies an 8x14 (diagonal) tile.
- Hadamard, R(π/4), delay are primitive.
- Controlled-Not from Controlled-R(π/4):

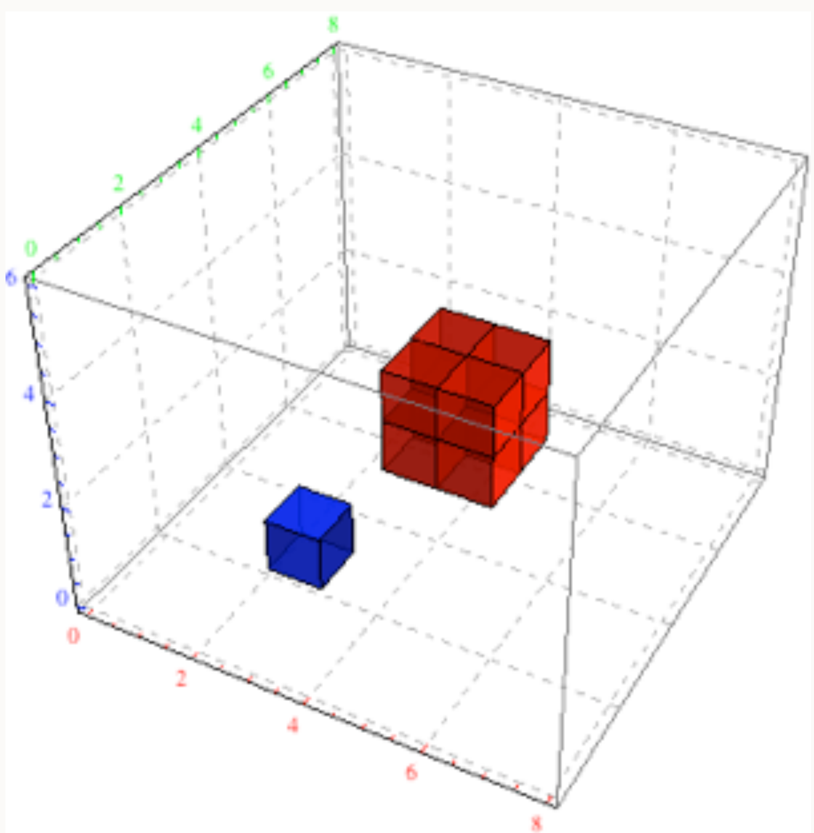


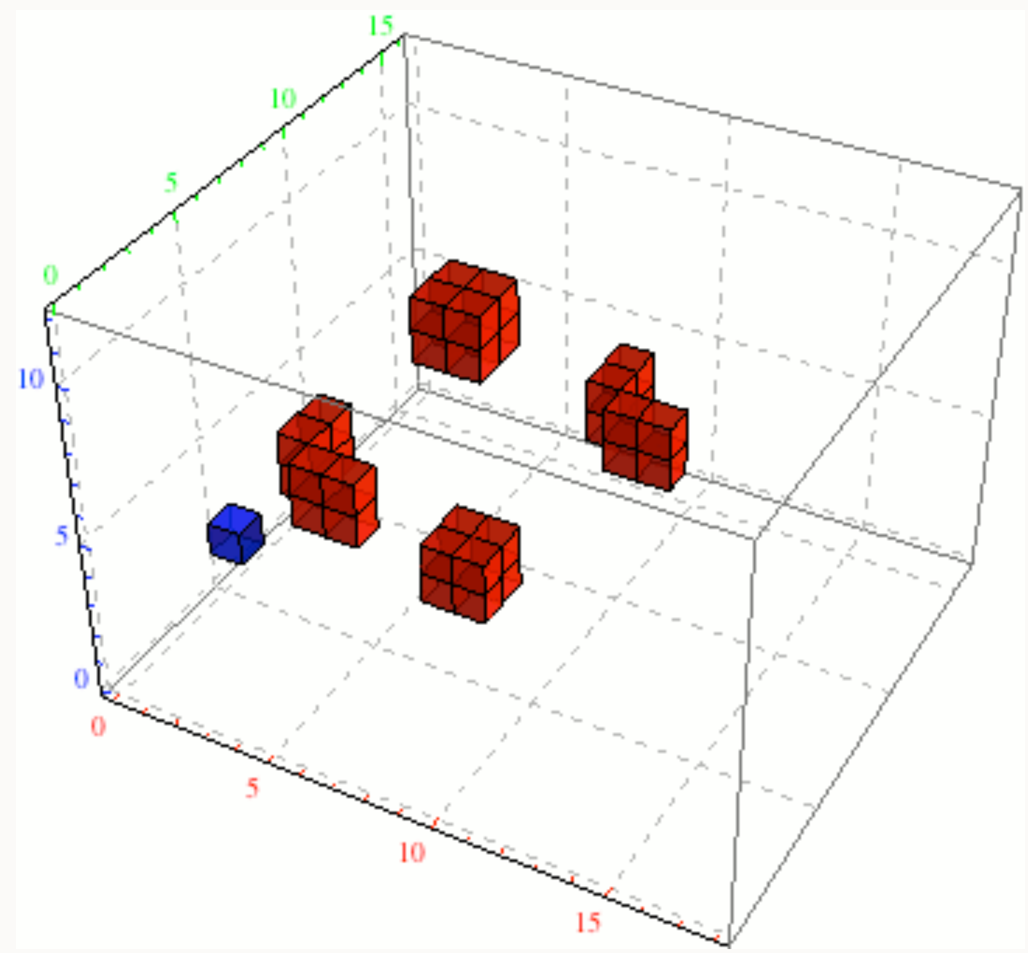
$$\text{cNot}\,|\psi\rangle = (\mathbb{I} \otimes H)(\text{cR}(\pi/4))^4(\mathbb{I} \otimes H)\,|\psi\rangle$$

- Barriers and delays complete wiring.

Minimal 3D PQCA

- *Binary* 3D PQCA
- More complicated, but seems interesting





Conclusion

- A simple PQCA capable of simulating all other ***QCA*** in a topology preserving manner.
- Simple, but not minimal.
- Step towards a *universal physical phenomenon*?
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